

# Letters

## Comments on "Attenuation Characteristics of Hollow Conducting Elliptical Waveguides"

G. FALCIASECCA, C. G. SOMEDA, AND F. VALDONI

In the above paper,<sup>1</sup> attenuation constants for several modes of a metal elliptical waveguide are computed by means of two basic formulas [footnote one, eqs. (1) and (4)]. These expressions do not coincide with those that Chu obtained long ago [1]. Several numerical discrepancies are pointed out.<sup>1</sup>

As Kretzschmar states, the standard first-order powerloss method for the attenuation in metal waveguides is very well known. As the partial steps are not reported in Kretzschmar's paper, it has to be inferred from the above-mentioned equations that the following quantity has been used as the real part of the wall impedance:

$$R = (\pi\mu f/\sigma)^{1/2} \quad (1)$$

$$\alpha_c \sqrt{a^3 \sigma} = \left( \frac{\pi \epsilon_0 \mu f_0}{1 - (f_c/f_0)^2} \right)^{1/2} C_{e_m}^2(\xi_0, q) \frac{\frac{4q}{e^2} \left( \frac{f_c}{f_0} \right)^2 (1 - e^2) \int_0^{2\pi} C_{e_m}^2(\eta, q) d\eta + [1 - (f_c/f_0)^2] \int_0^{2\pi} C_{e_m}^2(\eta, q) d\eta}{2\sqrt{1 - e^2} \int_0^{2\pi} \int_0^{\xi_0} [C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q) + C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q)] d\xi d\eta}.$$

where  $f$  is the frequency,  $\mu$  is the permittivity, and  $\sigma$  is the dc conductivity.

It is very well known that (1) holds in planar geometry and is also applicable to circular waveguides. It seems that the above-mentioned equations have been obtained by analogy with these cases.

A recent paper [2], which was published shortly after the date of Kretzschmar's original manuscript, follows a different path. The wave equation is solved for radial propagation in elliptical coordinates; then, as usual, the displacement current is neglected compared with the conduction current. Thus a longitudinal wall impedance

$$Z_s = (1 + j)Rb/h_t \quad (2)$$

and a transverse wall impedance

$$Z_\eta = (1 + j)Rb/h_t \quad (3)$$

are obtained, where  $2b$  is the minor axial length in the cross section of the elliptical waveguide and  $h_t$  is the first metric coefficient in the elliptical coordinate frame, evaluated on the metal wall. The details of the derivation are contained in [2].

From (2) and (3) one gets back to (1) if and only if the ellipse is indeed a circle, because then  $h_t = b$ . On the other hand, when the complete expressions (2) and (3) are introduced in the standard power-loss method, or in another first-order perturbation approach [3], then Chu's formulas [1] are obtained. Note that their original derivation had been performed by matching the fields on the elliptical boundaries.

Despite the little amount of experimental work that we are aware of, we trust Chu's formulas as being better grounded than those used by Kretzschmar. Therefore, we suggest that the very appreciable numerical evaluations of normalized attenuation charts, done in Kretzschmar's paper, be extended to those formulas and practical consequences of the different approach be pointed out.

### REFERENCES

- [1] L. J. Chu, "Electromagnetic waves in elliptic hollow pipes of metal," *J. Appl. Phys.*, vol. 9, pp. 583-591, Sept. 1938.
- [2] G. Falciasecca, C. G. Someda, and F. Valdoni, "Wall impedances and application to long-distance waveguides," *Alta Frequenza*, vol. 40, pp. 426-434, May 1971.
- [3] R. E. Colin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960, sect. 5.3.

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<sup>1</sup> J. G. Kretzschmar, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 280-284, Apr. 1972.

### Reply<sup>2</sup> by Jan G. Kretzschmar<sup>3</sup>

The only, but important, difference between Chu's formulas [1] and the ones given in [2] is obvious when the former are rewritten under the following normalized form.

For even TM modes:

$$\alpha_c \sqrt{a^3 \sigma} = \left( \frac{\pi \epsilon_0 \mu f_0}{1 - (f_c/f_0)^2} \right)^{1/2} \frac{C_{e_m}^2(\xi_0, q) \int_0^{2\pi} C_{e_m}^2(\eta, q) d\eta}{2\sqrt{1 - e^2} \int_0^{2\pi} \int_0^{\xi_0} [C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q) + C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q)] d\xi d\eta}.$$

For even TE modes:

$$\alpha_c \sqrt{a^3 \sigma} = \left( \frac{\pi \epsilon_0 \mu f_0}{1 - (f_c/f_0)^2} \right)^{1/2} \frac{\frac{4q}{e^2} \left( \frac{f_c}{f_0} \right)^2 (1 - e^2) \int_0^{2\pi} C_{e_m}^2(\eta, q) d\eta + [1 - (f_c/f_0)^2] \int_0^{2\pi} C_{e_m}^2(\eta, q) d\eta}{2\sqrt{1 - e^2} \int_0^{2\pi} \int_0^{\xi_0} [C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q) + C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q)] d\xi d\eta}.$$

It is now clear that the factor  $\text{th } \xi_0 = \sqrt{1 - e^2}$  in these equations replaces the factor  $\sqrt{1 - e^2} \cos^2 \eta$  in the corresponding formulas in [2]. This is due to the fact that the wall impedance has been taken equal to  $(\pi\mu f/\sigma)^{1/2}$ , as was pointed out by Falciasecca *et al.* A comparative study of both sets of formulas is indeed very interesting, and I hope to present the first results in the near future. Meanwhile, I would like to point out that it is not shown in [3] how the fields inside the elliptical waveguide and the fields in the metal wall can be matched at the boundary  $\xi = \xi_0$ . Another interesting problem is the accuracy of the asymptotic formulas for the modified Mathieu of the fourth kind and the approximations for their first derivative.

### REFERENCES

- [1] L. J. Chu, "Electromagnetic waves in elliptic hollow pipes of metal," *J. Appl. Phys.*, vol. 9, pp. 583-591, Sept. 1938.
- [2] J. G. Kretzschmar, "Attenuation characteristics of hollow conducting elliptical waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 280-284, Apr. 1972.
- [3] G. Falciasecca, C. G. Someda, and F. Valdoni, "Wall impedances and application to long-distance waveguides," *Alta Frequenza*, vol. 40, pp. 426-434, May 1971.

<sup>2</sup> Manuscript received August 7, 1972.

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## Focusing of 104-GHz Beams by Cylindrical Mirrors

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The use of cylindrical mirrors to focus 52-GHz beams over a 85-m-long path has been previously reported [1]. We shall report in this letter the results of tests made at 52 and 104 GHz over an extended 350-m-long path incorporating 10 refocusers (20 mirrors). The arrangement is shown in Fig. 1.

The round-trip loss measured in clear weather is 2.3 dB at 52 GHz and 1.0 dB at 104 GHz. These losses are exclusive of the launching and collecting-dish losses, the Mylar-window losses, and the absorption by the oxygen line of the atmosphere. The lower loss observed at 104 GHz can be accounted for by a reduced spillover at the

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